Parabolic Partial Differential Equations and Applications to Image Processing NW-PDEAIP (2015)

26-29 OCT 2015 @ PRASANTHI NILAYAM CAMPUS

The Department of Department of Mathematics and Computer Science, SSSIHL Prashanti Nilayam Campus, organized a four-day international workshop titled National Instructional Workshop on Parabolic Partial Differential Equations and Applications to Image Processing (NW-PDEAIP 2015) from 26 to 29 October 2015.

This workshop focused on very fundamental issues of theoretical interest regarding parabolic equations, discuss it's utility to develop models in different fields of applications. Parabolic equations are used in many application areas including image processing. Its theory is well developed. Anyone wanting to use these equations to model a real life problem needs to have a grasp of the fundamental theoretical issues related to this type of the equation. Error analysis based on Sobolev space methods discussed in the workshop can also be used while developing computational methods for parabolic equations to solve problems in image processing. The deliberations also included application case studies in the domain of image processing.

The workshop was planned in three parts:

1. Classical theory of Parabolic Equations
2. Sobolev spaces: to the extent required to support error analysis
3. Applications to Image processing: Modeling, Simulation and Analysis

The resource persons for the workshop were:

» Prof. Vasudeva Murthy, TIFR CAM Bangalore
» Prof. V Raghavendra, IIT Kanpur
» Dr. Venky Krishnan, TIFR CAM Bangalore
» Prof. S. Sundar, IITM

KEYNOTE TALKS

PROF. VENKY KRISHNAN, TIFR-CAM, BANGALORE

Microlocal Analysis in Image Processing

Prof. Venky Krishnan introduced three image processing problems: Ultrasound Reflection Tomography (URT), Synthetic Aperture Radar Imaging (SAR) and Intravascular Imaging (IVI). Ultrasound Reflection Tomography helps in the location and shape of a foreign object, such as tumours. Synthetic Aperture Radar Imaging is about finding an area from a height. Intravascular Imaging is about sending probes inside to find fat accumulations in the arteries. He then discussed exact methods for the above problems and about approximate methods to solve the above problems where he used microlocal analysis.

PROF. V RAGHAVENDRA, RTD. IITK, VISITING FACULTY SSSIHL

Foundations of PDEs for Image Processing

Professor Raghavendra started the talk by describing what is meant by a solution of a differential equation and why it is important to know what a solution is before we learn to actually find it. The solution of a general mth order linear partial differential equation is m times the continuously differentiable function which satisfies the given partial differential equation. He then motivated the need for distributions by drawing an analogy of root-finding for algebraic equations. Just like many equations over the field of reals such as x^2+1=0 do not possess any solutions in reals, many partial differential equations don't have any solution in space Cm. In the analogous case of root-finding of polynomials, field of real numbers is extended to algebraically closed field of complex numbers where every mth order polynomial has exactly m roots. Similarly to solve partial differential equations which do not possess solutions in space Cm we need to look for solutions in a larger space and this larger space turns out to be the space of distributions.

In the case of algebraic equations fundamental theorem of algebra guarantees existence of solutions, similarly Malgrange–Ehrenpreis theorem assures the existence of solution of a constant coefficient linear partial differential equation. Professor briefly mentioned about a celebrated example due to Hans Lewy known as "Lewy's Example" which is very fundamental to the theory of partial differential equations. Lewy's Example shows that above stated Malgrange–Ehrenpreis cannot be extended to linear partial differential equations with polynomial coefficients. Lewy's Example also demonstrate that the analog of the Cauchy–Kovalevskaya theorem does not hold in general if the analyticity condition is replaced by infinitely differentiable condition.

The professor then discussed the convolution operation of two functions which was followed by introduction to Sobolev Spaces. He then discussed many fundamental results in theory of Sobolev spaces including Friedrichs' theorem, Chain Rule in Sobolev space, theorem of Stampacchia and Poincare inequality. He gave overview of many proofs of above results. The series of talks concluded with questions and answer session.

PROF. A S VASUDEV MURTHY, TIFR-CAM, BANGALORE

Foundational Aspects of Linear Parabolic Equations

Prof. Vasudev Murthy spoke about the foundational aspects of Linear Parbolic Equations. He started with Heat and Wave equations and explained the utility of Fourier Transform in solving the equations. Stating the different methodologies of determining solutions such as the exact solutions obtained by Fourier Transform, considering a sequence of auxiliary problems which successively approximate the equation and establishing estimates. A detailed view of dissipative and conservative nature of equations was dealt with. The well posedness problem of existence, uniqueness and continuous dependency of solutions on initial data associated with parabolic equations was also dealt with considering three types of boundary conditions namely Dirichlet, Neuman and Robin. The erudite speaker has then discussed intuitive aspects of the diffusion processes such as convolution with Gaussian kernel and averaging processes. Finally, a cursory view of Finite Elemental Methods was given.