

# Ph.D. Programme in Physics

## Model Question Paper

### RESEARCH APTITUDE ASSESSMENT TEST

Time : 2 Hours

Max. Marks : 75

#### I. Part A: Multiple Choice Questions

- 30 marks

Choose the correct Response, viz., A, B, C, D or E for the Questions from 1 - 30 which carry ONE mark each. Please NOTE that an **incorrect response** will attract **negative marking**. (For Multiple Choice questions with 5 options,  $\frac{1}{4}$ <sup>th</sup> mark will be deducted for an incorrect answer.)

#### Questions 1-10 carry ONE mark each

1. For a particle in an infinite potential well, let  $\psi_n(x)$  be the wave function. In the middle of the well, the probability density vanishes for

- (A) The ground state only ( $n=1$ )                      (B) States of even  $n$  ( $n=2, 4, 6, \dots$ )  
(C) States of odd  $n$  ( $n=1, 3, 5, \dots$ )                      (D) All states ( $n=1, 2, 3, \dots$ )  
(E) All states except the ground state

( \_\_\_ )

2. A system is in a state  $(5|1, 1 \rangle + 3|5, 1 \rangle + 2|5, -1 \rangle)/\sqrt{38}$  where the states are angular momentum kets  $|l, m \rangle$ ; i.e., the eigenvalues of  $L^2$  are  $l(l+1)\hbar^2$  and those of  $L_z$  are  $m\hbar$ . The probability of obtaining the result  $l = 5\hbar$  is

- (A) 36/1444                      (B) 9/38                      (C) 13/38                      (D)  $5/\sqrt{38}$                       (E) 34/38

( \_\_\_ )

3. The first three energy levels of  $^{228}\text{Th}$  are shown below

$4^+$	_____	187keV
$2^+$	_____	57.5keV
$0^+$	_____	0keV

The expected spin-parity and energy of the next level are given by

- (A)  $6^+$ ; 400 keV                      (B)  $6^+$ ; 300 keV                      (C)  $2^+$ ; 400 keV  
(D)  $4^+$ ; 300 keV                      (E) None of the above

( \_\_\_ )

4. The ground state of  $^{207}\text{Pb}$  ( $Z=72$ ) nucleus has spin-parity  $J^\pi = 1/2^-$ , while the first excited state has  $J^\pi = 5/2^-$ . The electromagnetic radiation emitted when the nucleus makes a transition from the first excited state to ground state will have multipolarities of
- (A) E2 and E3                      (B) M2 and E3                      (C) E2 and M3  
 (D) M2 and M3                      (E) cannot be predicted
- ( \_\_\_ )
5. The root mean square velocity of a molecule of mercury vapour at 300K, given molecular weight of mercury = 221 g/mol, is (in cm/s)
- (A)  $1.93 \times 10^4$                       (B)  $1.93 \times 10^6$                       (C)  $1.93 \times 10^8$   
 (D)  $3.72 \times 10^4$                       (E)  $3.72 \times 10^8$
- ( \_\_\_ )
6. Which of the following relation is true for an ideal gas? (T is temperature, E is modulus of isothermal elasticity,  $\alpha$  is the coefficient of volume expansion)
- (A)  $C_p - C_v = T E \alpha^2 V$                       (B)  $C_p - C_v = T E \alpha^2 V^2$                       (C)  $C_p - C_v = -TE\alpha V$   
 (D)  $C_p - C_v = -TE \alpha^2 / V$                       (E)  $C_p - C_v = T^2 E \alpha^{1/2} V$
- ( \_\_\_ )
7. A circularly polarized light on reflection from a mirror becomes a
- (A) Linearly polarized light parallel to the plane of reflection  
 (B) Linearly polarized light perpendicular to the plane of reflection  
 (C) Elliptically polarized light  
 (D) Unpolarized light  
 (E) None of the above
- ( \_\_\_ )
8. Two lenses are made from the same material. The lens with a longer focal length is relatively
- (A) Thick                      (B) Thin                      (C) not strongly curved  
 (D) both B and C                      (E) both A and C
- ( \_\_\_ )

9. Consider X-ray diffraction from a crystal with a *fcc* lattice. The lattice plane for which there is NO diffraction peak is  
 (A) (2,1,2) (B) (1,1,1) (C) (2,0,0)  
 (D) (3,1,1) (E) None of the above  
 ( \_\_\_ )

10. Consider an anti-symmetric tensor  $P_{ij}$  with indices  $i$  and  $j$  running from 1 to 5. The number of independent components of the tensor is  
 (A) 3 (B) 10 (C) 9 (D) 6 (E) 5  
 ( \_\_\_ )

**Questions 11-20 carry TWO marks each**

11. A particle of mass  $m$  has wave function  $\psi(x, t) = e^{i\omega t}(\alpha \cos kx + \beta \sin kx)$ . The probability current density is  
 (A) 0 (B)  $\frac{\hbar k}{m}$  (C)  $\frac{\hbar k}{2m}(|\alpha|^2 + |\beta|^2)$   
 (D)  $\frac{\hbar k}{m}(|\alpha|^2 - |\beta|^2)$  (E)  $\frac{\hbar k}{2mi}(\alpha^* \beta - \beta^* \alpha)$   
 ( \_\_\_ )

12. In deep inelastic scattering electrons are scattered off protons to determine if a proton has any internal structure. The energy of the electron for this must be at least  
 (A)  $1.25 \times 10^9 \text{ eV}$  (B)  $1.25 \times 10^{12} \text{ eV}$  (C)  $1.25 \times 10^6 \text{ eV}$   
 (D)  $1.25 \times 10^8 \text{ eV}$  (E)  $1.25 \times 10^{10} \text{ eV}$   
 ( \_\_\_ )

13. Let  $v$ ,  $p$  and  $E$  denote the speed, the magnitude of momentum and the energy of a free particle of rest mass  $m$ . Then  
 (A)  $\frac{dE}{dp} = \text{const}$  (B)  $p = mv$  (C)  $v = \frac{cp}{\sqrt{p^2 + m^2 c^2}}$   
 (D)  $E = mc^2$  (E) none of the above  
 ( \_\_\_ )

14. Three particles of equal mass  $m$  are connected by two identical massless springs of stiffness

constant  $k$ . If  $x_1, x_2$  and  $x_3$  denote the horizontal displacements of the masses from their respective equilibrium positions, the potential energy of the system is

- (A)  $\frac{1}{2}k[x_1^2 + x_2^2 + x_3^2]$
  - (B)  $\frac{1}{2}k[x_1^2 + x_2^2 + x_3^2 - x_2(x_1 + x_3)]$
  - (C)  $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 + 2x_2(x_1 + x_3)]$
  - (D)  $\frac{1}{2}k[x_1^2 + 2x_2^2 + x_3^2 - 2x_2(x_1 + x_3)]$
  - (E)  $\frac{1}{2}k[x_1^2 + x_2^2 + 3x_3^2 + x_2(x_1 + x_3)]$
- ( \_\_\_ )

15. The donor concentration in a sample of  $n$ -type silicon is increased by a factor of 100. The shift in the position of the Fermi level at 300K, assuming the sample to be non-degenerate is \_\_\_  $meV$ .

- (A) 10
  - (B) 100
  - (C) 115
  - (D) 150
  - (E) 200
- ( \_\_\_ )

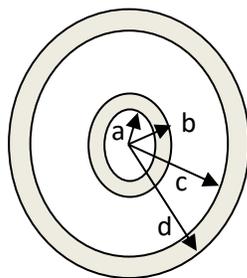
16. to 20. ...

### Part - B

(45 Marks)

**II. Write any NINE questions from the following in the sheets provided with the question paper. Each question carries FIVE marks.**

1. Suppose that a system  $S$  has the single coordinate  $q$  and Lagrangian  $L = \frac{\dot{q}^2}{4} - \frac{q^2}{9}$ . Find the paths in Hamiltonian phase space that corresponds to the motion of  $S$ .
2. The magnetic field inside a long cylindrical conductor of radius  $a = 0.1m$  is  $\vec{H} = \frac{10^4}{r} \left( \frac{1}{\alpha^2} \sin(\alpha r) - \frac{r}{\alpha} \cos(\alpha r) \right) \hat{\phi}$ , where  $\alpha = \frac{\pi}{2a}$ . Determine the total current in the conductor.
3. Consider the two concentric conducting spherical shells of inner and outer radius  $a, b$  and  $c, d$  respectively as shown in the figure. Both shells are given a total positive charge  $Q$  each. Determine the electric field in the regions i)  $a < r < b$  ii)  $b < r < c$  and iii)  $r > d$ . Also determine the charge density on the outer surface of the outer shell.



4. Consider a Fabry-Perot cavity, the separation between the mirrors is 10 cm. If a beam having a central frequency of  $\nu_0 = 6 \times 10^{14}$  Hz and a spectral width of 7000 M Hz, calculate the spacing of the adjacent modes, and the frequencies of the output beam.
5. In a double slit experiment the distance between slits is  $d=5.0$  mm and the distance to the screen is  $D=1.0$  m. There are two interference patterns on the screen: one due to light with  $\lambda_1= 480$  nm and another due to light with  $\lambda_2= 600$  nm. What is the separation between third order ( $m=3$ ) bright fringes of the two patterns?
6. At  $t=0$ , the wave function of a particle inside an infinite square well potential of width  $a$  is given by  $\Psi(x, 0) = A(\Psi_1(x) + \Psi_2(x))$ , where  $\Psi_1$  and  $\Psi_2$  are normalized ground state and first excited state eigenfunctions. Find A. What is the probability density  $P(x,t)$  at a later time  $t$ ?
7. to 12. ...

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