

Ph.D. Programme in Mathematics

Model Question Paper

RESEARCH APTITUDE ASSESSMENT TEST

Time : 2 Hours

Max. Marks : 75

I. Part A: Multiple Choice Questions 30 x 1 mark = 30 marks

Choose the correct Response viz., A, B, C, D or E for the Questions from 1 - 30 which carry ONE mark each. Please NOTE that an **incorrect response** will attract **negative marking**. (For Multiple Choice question with 5 options, $\frac{1}{4}$ th mark shall be deducted for an incorrect answer.)

1. Consider the following system of equations:

$$\begin{aligned}x_1 + x_3 &= 3 \\x_1 - x_2 - x_3 &= 1 \\-x_1 + x_2 &= 4\end{aligned}$$

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The above system of linear equations is:

- A) consistent with infinitely many solutions
- B) consistent with a unique solution
- C) inconsistent
- D) inconsistent but has many solutions
- E) inconsistent but has a unique solution

2. The eigen values of a skew-symmetric matrix are

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- A) all zeros
- B) always real
- C) always purely imaginary
- D) always zero and purely imaginary
- E) does not always exist

3. The rank of the matrix $\begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$ is

()

- A) 0
- B) 1
- C) 2
- D) 3
- E) 4

4. Which of the following is a linearly independent set of vectors over \mathbb{R} ?

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- A) $\{\}$
- B) $\{0\}$
- C) $\{1,2\}$
- D) $\{(1,1), (2,2)\}$
- E) $\{(0,0), (1,1)\}$

5. Given that on tossing two fair coins one head appears. What is the probability that head appears on the other coin as well? ()
 A) $\frac{1}{2}$ B) $\frac{1}{3}$ C) $\frac{1}{4}$ D) $\frac{1}{6}$ E) cannot be determined
6. If E denotes expectation, the variance of a random variable X is given by ()
 A) $E[X^2] - E^2[X]$ B) $E[X^2] + E^2[X]$ C) $E[X^2]$
 D) $E^2[X]$ E) $E[X^2 + E^2[X]]$
7. Which one of the following number systems is algebraically closed? ()
 A) Natural numbers B) Integers C) Real
 D) Complex E) Quaternions
8. Which of the fields is a degree 2 extension of \mathbb{R} ? ()
 A) $\mathbb{Q}(\sqrt{2})$ B) $\mathbb{Q}(i)$ C) \mathbb{R} D) \mathbb{C} E) None of the above
9. Which one of the following maps on the complex plane is NOT analytic? ()
 A) constant B) scaling C) translation
 D) rotation E) reflection about real axis
10. What is the order of the Dihedral group D_5 ? ()
 A) 5 B) 10 C) 20 D) 25 E) None of the above
11. Which of the following spaces is not separable? ()
 A) c B) ℓ^1 C) ℓ^2 D) ℓ^∞ E) None of the above
12. Which of the following is weakly convergent but not strongly? ()
 A) $\{e_n: n \in \mathbb{N}\}$ B) $\{(\xi_n^k): \xi_n^k = 1, n \in \mathbb{N}\}$
 C) $\{(\xi_n^k): \xi_n^k = \xi_n, n \in \mathbb{N}\}$ D) $\{(\xi_n^k): \xi_n^k = \xi_n + \frac{1}{n}, n \in \mathbb{N}\}$
 E) None of the above
13. The residue at the $z = 0$ for the function $f(z) = \frac{1}{z^2+z}$ is given by ____ ()
 A) 1 B) 0 C) πi D) $2\pi i$ E) does not exist
14. Classify the following partial differential equation: ()
 $u_x + u_y + u^2 = f(x)$
 A) linear B) semi-linear C) quasi-linear
 D) fully nonlinear E) none of the above

15. Which one of the following is true about the solution of the following initial value problem? ()

$$y' = y^{-2}(2 - 3x), \quad y(0) = 19:$$

- A) Non-existent B) Trivial C) Infinite
D) Unique E) Vacuous

16. to 30. ...

Part - B

II. Answer any 9 of the following in about 150 words each in the sheets provided with the question paper:

(9 x 5 = 45 marks)

1. Show that the transformation $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$ defined by $T(x, y, z) = (0, x, y)$ is not diagonalizable by quoting relevant results.
2. Is the function $f(x) = \begin{cases} x \sin 1/x & x \neq 0 \\ 0 & x = 0 \end{cases}$ differentiable at $x = 0$? Justify your answer.
3. Show using induction that $1 + 3 + 5 + \dots + (2n - 1)$ is a square for $n \in \mathbb{N}$.
4. Determine the radius of convergence of the series $\sum_{k=1}^{\infty} \frac{z^{2k}}{4^k k^k}$.
5. Write the iterative equation to solve the polynomial equation $x^3 + 4x - 9 = 0$ numerically using Newton-Raphson Method.
6. Show that in an inner product space over the reals \mathbb{R} , two non-zero vectors are orthogonal if they satisfy the Pythagoras theorem.
7. to 12.

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